

**Simulation Based  
Engineering  
Science Series**

**KINETICS OF MACHINERY  
THROUGH HYPERWORKS**

**Professor J. S. Rao**

**President**

**The Vibration Institute of India**

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**Krishna**

# **KINETICS OF MACHINERY THROUGH HYPERWORKS**

by

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# Preface

This book should be read after *Kinematics of Machinery* through HyperWorks, by the same author from Springer, published in 2011. There the subject of Kinematics was studied that has origins in Science Revolution and Calculus of 18th century from Sir Isaac Newton. In *Kinematics* as well as *Kinetics* of the present book only rigid bodies including particles were accounted. No body is actually rigid; it is a concept. The flexibility effects become important when the body is subjected to alternating forces with a frequency that is near or beyond the natural frequency of the flexible body. Therefore this subject of flexible body dynamics (Vibrations) comes after *Kinematics* and *Kinetics* study of rigid bodies.

The industrial revolution itself started as an off shoot from Science revolution. Man always wanted to get his chores done faster and better than what he can do himself; so he began training animals to do this labor, e.g., bullock carts and horse drawn vehicles. Nearly two millennia ago in Alexandria, Hero invented a reaction steam turbine; but apparently he had no idea of what to do with this as no application is known without science. Man invented hydropower, wind sails, gun powder etc. prior to the science and industrial revolutions. Thomas Savery's pumping engines in the year 1698 were probably the first real applications but lacked science. It is Joseph Black's discovery in 1761 at the beginning of Science Revolution on Latent Heat that ushered the subject of Thermodynamics and applied by James Watt to begin industrial revolution and reciprocating steam engines as we know today. The reciprocating steam engine is slow speed one and bulky; design based on stress rarely entered as a critical factor.

The steam turbine of de Laval and Generator of Thomas Alva Edison in early 1880s ushered design as we know today. Once man has learnt of electricity, his urge to design and build higher capacity, speed and efficient machines became an immediate need. By 1930's Timoshenko school brought in an approximate engineering approach, initially graphical followed by analytical methods of design. With the advent of ENIAC in Philadelphia during the second world war changed the scenario completely and the computers have replaced log tables and slide rules in computation. The present day designs are all based on faster and more accurate digital means and that is why the subject orientation has changed. The teaching methods have to be appropriately reoriented, the subject remaining the same. This is the reason why this book is written after the *Kinematics* book dealing with *Kinetics*.

In this process two other books are written that can be followed using Science to Engineering Approach. They are: 1. *Simulation Based Engineering in Fluid Flow Design* and 2. *Simulation Based Engineering in Solid Mechanics* both from Springer, 2017. It is also advisable that one reads *History of*

Rotating Machinery Dynamics book from Springer in 2011 to have a clear background on the developments from the ending of ice age 15 millennia ago.

There are several books available on the same subject; one may like to refer to them as the subject matter is same.

**J. S. Rao**

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## Static Force Analysis and Friction

**Summary.** In this chapter we begin with static forces that are capable of producing motion in a linkage or a machine member. We will consider both sliding and turning pairs with dry friction. Next some simple devices using friction, like a screw and nut, brake, belt drive are considered. Finally rolling element bearings with dry friction and fluid film friction in journal bearings are considered.

We have studied different mechanisms and their motion in the subject of Kinematics. There we did not question what causes the motion? There has to be a force to move the rigid bodies or links of the mechanism. This force comes from an energy source external combustion or internal combustion. In an internal combustion engine the fuel is burnt according to a periodic cycle; this applies a force on one of the bodies, e.g. piston in a cylinder. This force is transmitted continuously throughout the period and repeats itself.

Besides the applied forces, we have friction between the moving bodies and in addition the inertia force due to motion itself from Newton's law. These forces vary depending on the motion periodically. So in design analysis we determine these forces transmitted from member to member and determine their influence on the rigid bodies of the linkage in a continuous manner over the period. Continuity in practice means 5 or 10 degrees of rotation in a graphical analysis or 0.005 or 0.01 seconds or even lower by a computer over which the analysis is carried out. We can then make a table or a plot of these forces over a period. Eventually we utilize these forces to determine the stresses and member sizes using solid mechanics. In this chapter we will study only forces that are applied as static forces at a given instance of time or the location at which the rigid bodies make up the linkage.

IFToMM (The International Federation for the Promotion of Mechanism and Machine Science) is a world body established in 1969 for promotion of Mechanisms and Machines Science. One of its tasks is on standardization of Mechanism and Machine Science terminology which resulted in a concise publication in 2003 covering various aspects of Mechanisms and Machines.

In this book, where necessary all definitions are taken from these terminology standards<sup>1</sup>.

**Statics:**

Branch of theoretical mechanics dealing with the equilibrium of bodies under the action of forces.

**Force:**

Action of its surroundings on a body tending to change its state of rest or motion.

**Line of Action of Force:**

Straight line along which the vector representing a given force lies.

**Magnitude of Force:**

Number of units of force obtained by comparing a given force with a standard, taken as unit force.

**Static [Active, Applied] Force:**

Force capable of producing motion

**Reaction:**

Force arising in a constraint and acting upon a constrained body due to the action of an active force upon that body.

**Normal Reaction:**

Component of reaction perpendicular to the surface of the body.

**Tangential Reaction:**

Component of reaction tangential to the surface of a body.

---

<sup>1</sup> IFToMM Commission A, Terminology for the Mechanism and Machine Science, Mechanism and Machine Theory, Vol. 38, Nos. 7–10, pp. 598–1111, 2003.

**External Force:**

Force due to the action of another body or system on the body or system under consideration.

**Internal Force:**

Force acting upon a particle or a set of particles of a given system, originating from another particle or set of particles in the same system.

**Elastic Force:**

Internal force arising in an elastically strained body.

**Concentrated Force:**

Force whose action may be regarded as being applied at a point.

**Distributed [Continuous] Force:**

Force that is spread along a line or over a surface.

**Body Force:**

Force which acts on the elements of the volume of a body.

**Surface [Traction] Force:**

Force whose action is distributed over the surface or part of the surface of a body.

**Compressive Force:**

Normal component of a force that acts on the surface of a body and which is directed into the body.

**Tensile Force:**

Normal component of a force that acts on the surface of a body and which is directed out from the body

**Axial [Longitudinal] Force:**

Force that acts normal to a given cross-section of a bar and through its centroid.

**Shear [Transverse] Force:**

Force that acts normal to the central axis of a bar.

**Bearing Force:**

Action of one link of a mechanism upon another at a bearing.

**Parallel Force System:**

Set of forces whose lines of action are parallel.

**Coplanar Force System:**

Set of forces whose lines of action lie in one plane.

**Concurrent Force System:**

Set of forces whose lines of action intersect each other at one point.

**Spatial Forces System:**

Set of forces whose lines of action do not lie in one plane.

**Resultant Force:**

Vector sum of a set of forces.

**Moment of a Force about an Axis:**

Component along a given axis of the moment of a force about any point on the axis.

**Moment of a Force about a Point:**

Vector product of a radius vector from the point to the line of action of the force and the force itself.

**Moment Arm:**

Shortest distance to the line of action of a force from a given point.

**Couple:**

Pair of parallel forces that are equal in magnitude, but opposite in sense; Vector moment of two parallel forces that are equal in magnitude but opposite in sense.

**Moment of a Couple:**

Vector sum of the moments about any point in space of the forces that form a given couple.

**Resultant Moment:**

Moment equal to the vector sum of the moments of all the forces of a system about a chosen point.

**Torsional Moment [Twisting Moment, Torque]:**

Component normal to the plane of cross-section of a bar of the moments about the centroid of the forces acting on the cross-section.

**Input Torque:**

Torque applied to driving (or input) link of a mechanism.

**Output Torque:**

Torque applied by the output link of a mechanism.

**Equilibrium:**

State of a system of forces and couples when the resultant force and the resultant couple of the system are simultaneously zero.

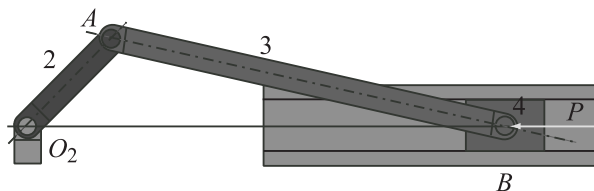
**Load:**

Set of active forces acting upon a body or system.

## 1.1 Reciprocating Engine Mechanism

Fig. 1.1 shows the reciprocating engine mechanism with the applied force (Gas force)  $P$ , on link 4 (piston). Here we show only one position of the linkage to illustrate the process of the forces received by other links due to application of the gas force  $P$ .

The free body diagram of link 4 is shown in Fig. 1.2a, wherein the applied force  $P$  and the reaction forces from its neighboring links 1 and 3 are shown.

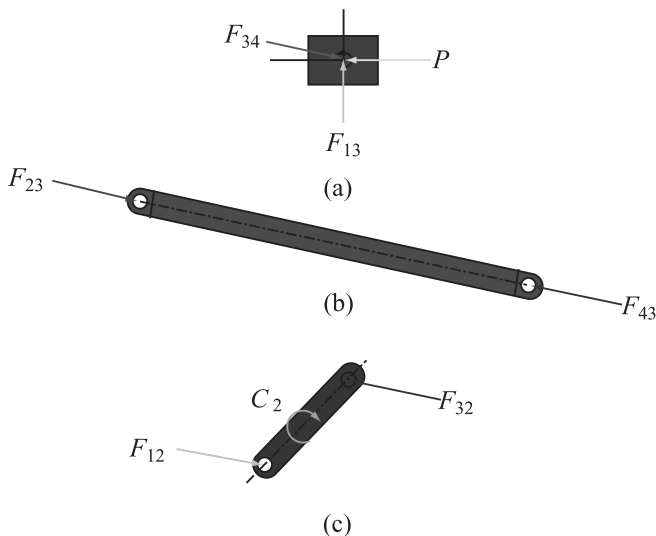


**Figure 1.1.** A Reciprocating Engine Mechanism

Note the subscript marked for the reaction forces, the first subscript denotes the link from which the reaction is applied and the second subscript, the link under consideration.  $F_{14}$  is the reaction from the cylinder walls on to the piston. In absence of friction, this reaction is a normal reaction force. Its line of action is known, however, its direction is unknown for the moment. The reaction force  $F_{34}$  from the connecting rod to the piston is shown along the connecting rod line itself, as it will be evident from the next diagram.

The connecting rod link 3, has no applied forces, therefore it has only reaction forces from links 2 and 4, see Fig. 1.2b. Two forces can keep a body in equilibrium, only when they are equal, opposite and collinear. Hence, the lines of action of reaction forces  $F_{43}$  and  $F_{23}$  are along the link 3. This is why, the reaction force  $F_{34} = -F_{43}$  is marked along the connecting rod in Fig. 1.2a.

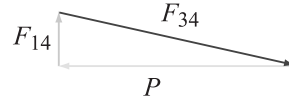
The equilibrium diagram of the crank, link 2, is shown in Fig. 1.2c. There are two reaction forces here  $F_{32}$  and  $F_{12}$  and we know that the crank is delivering torque (output torque). It is evident that  $F_{32}$  should be equal and opposite to  $F_{23}$  and therefore  $F_{12}$  should be equal and opposite of  $F_{32}$ . The



**Figure 1.2.** Free Body Diagrams of Links 4, 3 and 2

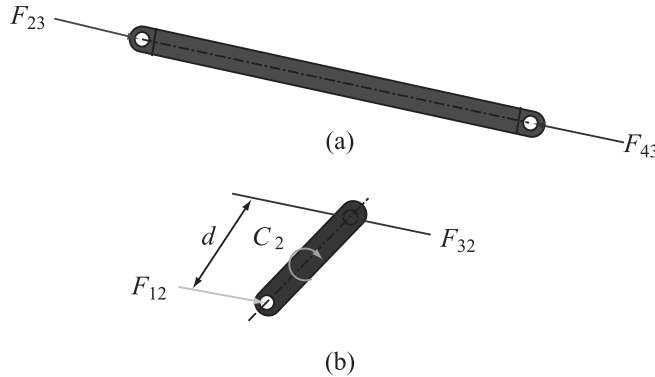
couple formed by the two equal and opposite forces  $F_{32}$  and  $F_{12}$  is the output torque and will be equal and opposite of the load torque.

Fig. 1.3 shows the force polygon of link 4,  $P + \rightarrow F_{14} + \rightarrow F_{34} = 0$  from which the unknowns  $F_{14}$  and  $F_{34}$ , their magnitudes and directions are obtained.



**Figure 1.3.** Force Polygon of Link 4

Fig. 1.4a shows the actual directions of the forces  $F_{43}$  and  $F_{23}$ . The force diagram of link 2 is shown in Fig. 1.4b.  $d$  is the moment arm and  $C_2 (= F_{32}d)$  is the resisting (load) torque that keeps the body in equilibrium.



**Figure 1.4.** Equilibrium Diagrams of Links 3 and 2

## 1.2 Quick Return Mechanism

Fig. 1.5 shows the configuration of a quick return motion mechanism. The cutting force  $P$  is acting on the tool which is rigidly connected to link 6.

Link 6 is shown in Fig. 1.6a. From the equilibrium diagram of link 5, Fig. 1.7, which is a two force member, we deduce that the reaction force from link 5 to link 6,  $F_{56}$  is in the direction of link 5. Here we are confronted with a case where the three forces,  $P$ ,  $F_{56}$  and  $F_{16}$  are not concurrent. Actually, the slider receives two reaction forces from link 1, one to the left  $F_{16}^L$  and the other to the right  $F_{16}^R$  of link 6 as shown in Fig. 1.6a, such a force system is known as cocking action. It is however difficult to say at which corner,  $C_1$ ,  $C_2$ ,  $C_3$  or  $C_4$  these two forces are acting.

This can be ascertained from the force polygon of link 6 shown in Fig. 1.6b. First assume that the forces  $F_{16}^L$  and  $P$  are replaced by their resultant which acts at the intersection point of these two forces. Similarly, the two forces  $F_{16}^R$



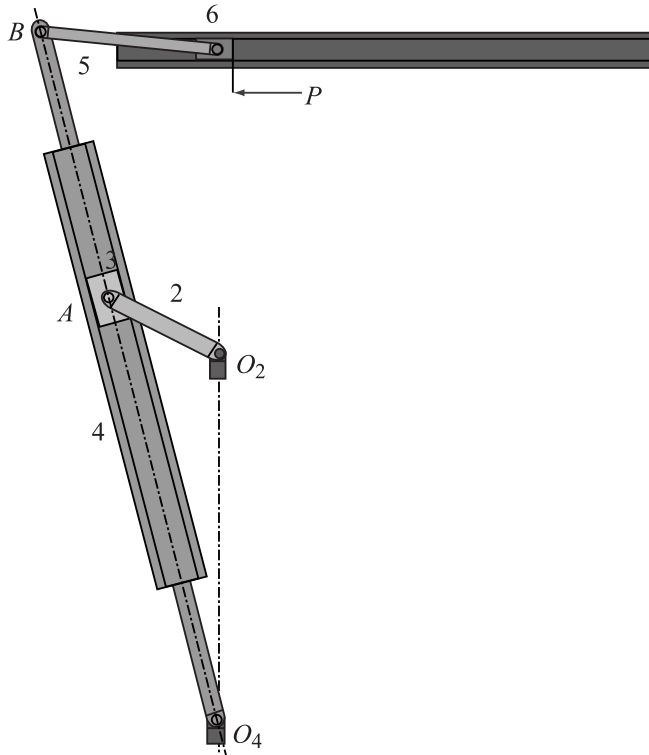


Figure 1.5. Quick Return Motion Mechanism

and  $F_{56}$  are replaced by their resultant acting at their intersection point as shown in Fig. 1.6a. The two resultant forces thus obtained keep the link 6 in equilibrium. Since there are only two forces keeping the body in equilibrium, it is obvious that these two forces are collinear, equal and opposite on the dashed line shown in the figure.

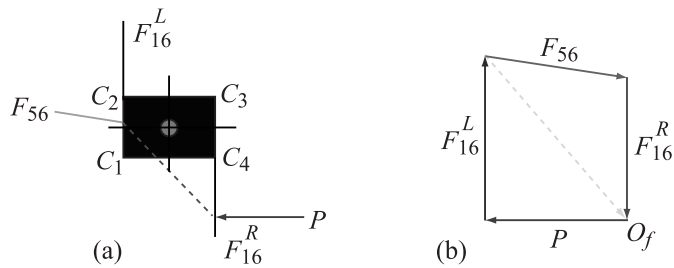


Figure 1.6. Equilibrium of Link 6

First set off to a suitable scale, the known force  $P$ . Draw a vertical line to represent the reaction force  $F_{16}^L$  at the tip of the force  $P$  and another parallel to the dashed line in Fig. 1.6a to represent their resultant. The intersection point defines the force  $F_{16}^L$ . It is now easy to complete the rest of the force polygon as shown in Fig. 1.6b and determine  $F_{16}^L$ ,  $F_{16}^R$  and  $F_{56}$ . Since  $F_{16}^L$  is vertically upwards, the contact on the left hand side takes place at  $C_1$  and similarly, we find that the contact at right hand side takes place at  $C_3$ .

From Fig. 1.7, we obtain  $F_{45} = -F_{65}$ . Further,  $F_{54} = -F_{45}$ . Now, we are in a position to draw the equilibrium diagram of link 4 as shown in Fig. 1.8a. The reaction force  $F_{34}$  is normal to the link 4 and link 3 passing through the hinge at  $A$ . The reaction force from the ground link to link 4 passes through the bearing at  $O_4$  as shown. The direction of this force can be found from the fact that if there are three forces keeping a body in equilibrium, they all should be concurrent.

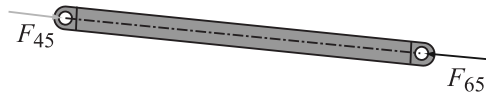


Figure 1.7. Equilibrium of Link 5

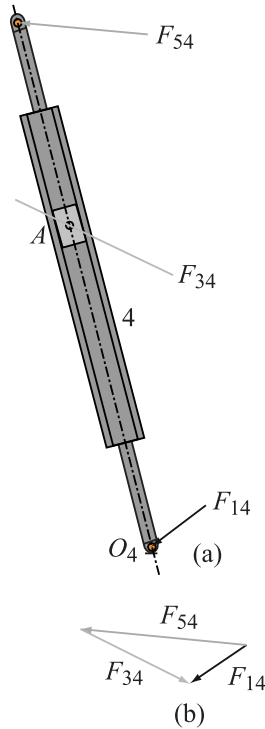


Figure 1.8. Quick Return Motion Mechanism

Therefore, first find the intersection point of the forces  $F_{54}$  and  $F_{34}$  and join this point with the hinge at  $O_4$  to obtain the direction of force  $F_{14}$  then complete the solution with the force polygon drawn in Fig. 1.8b.

The free body diagrams of links 3 and 2 are given in Figs. 1.9 and 1.10 respectively. The driving torque from the motor is  $C_2 = F_{32}d$ .

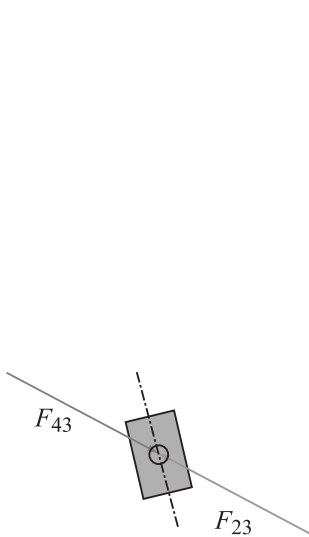


Figure 1.9. Equilibrium of Link 3

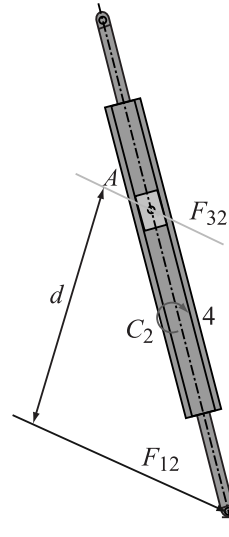


Figure 1.10. Equilibrium of Link 2

### 1.3 Four Link Mechanism

The static force analysis of a four link mechanism is illustrated here. Links 3 and 4 experience two static forces  $F_3$  and  $F_4$  acting at points  $Q$  and  $P$  respectively. We have to determine what force  $F_2$  with the line action shown in Fig. 1.11 acting at point  $S$  will keep the linkage in equilibrium.

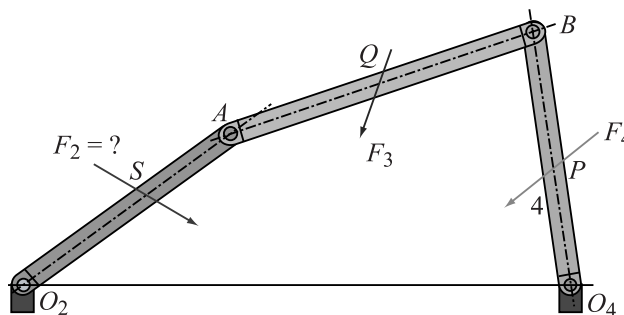


Figure 1.11. Four Link Mechanism

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